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Infinite statistics, symmetry breaking and combinatorial hierarchy

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Abstract

The physics of symmetry breaking in theories with strongly interacting quanta obeying infinite (quantum Boltzmann) statistics known as quons is discussed. The picture of Bose/Fermi particles as low energy excitations over nontrivial quon condensate is advocated. Using induced gravity arguments it is demonstrated that the Planck mass in such low energy effective theory can be factorially (in number of degrees of freedom) larger than its true ultraviolet cutoff. Thus, the assumption that statistics of relevant high energy excitations is neither Bose nor Fermi but infinite can remove the hierarchy problem without necessity to introduce any artificially large numbers. Quantum mechanical model illustrating this scenario is presented.

1 Introduction

A truly remarkable phenomenon in Nature is coexistence of a few vastly different scales in one theoretical framework. Sometimes it is known as a problem of "large numbers" [1]. The best known and physically the most interesting example of this kind is given by the Standard Model. As is well known this theory contains a tower of scales starting from electron mass $m_e = 511\,\mathrm{keV}/c^2$ and going up to QCD scale $m_p = 938\,\mathrm{MeV}/c^2$, weak scale $m_W = 80\,\mathrm{GeV}/c^2$ and finally ending by the Planck scale $M_P = 1.2 \cdot 10^{19}$ GeV$/c^2$ which represents, as many physicists believe, the ultimate ultraviolet edge of our world.[1] Despite each large ratio in this tower calls for explanation, of particular interest is the parameter M_P in the weak scale units, which is given by the number of order 10^{17}. The danger this large number is for the stability of radiative corrections to Higgs boson mass has been widely discussed in the literature (see recent review [2]).

An interesting approach to the hierarchy problem known as TeV-scale gravity and extra-dimensions scenarios came about a decade ago. These models assume that the geometric properties of our familiar (3+1) dimensional space-time change at some scale L, which is

[1] We leave aside the lower part of this tower, corresponding to smaller energies relevant for condensed matter physics, chemistry and biology.

supposed to be much larger than $L_P = 1/M_P$ and perhaps of (1-2 TeV)$^{-1}$ range. This change can be accompanied by appearance of some additional particles, for example of the Kaluza-Klein type. Among attractive features of these models is emergent nature of the Planck scale L_P. In the original proposal [3, 4] the fundamental ratio between L_P and L takes the form

$$\left(\frac{L}{L_P}\right)^2 \sim \frac{V_n}{L^n} \tag{1}$$

where L is electroweak scale, n - number of compact extra dimensions and V_n - their volume. Soft SM fields propagates only in 4 dimensions, while gravitons feel the full $4+n$ dimensional geometry. In this scenario the smallness of the ratio L/L_P follows from the large (in units of L, i.e. sub-millimeter for the most phenomenologically interesting case $n=2$) size of extra dimensions. The picture suggested in [5, 6] is different: the model with one warped extra dimension generates the hierarchy

$$\log \frac{L}{L_P} \sim \frac{\pi r_c}{L_P^{(5)}} \tag{2}$$

where πr_c is the size of compact extra dimension and $L_P^{(5)}$ - fundamental five dimensional Planck length. The logarithmic function maps huge hierarchy between L and L_P into much weaker hierarchy between r_c and $L_P^{(5)}$. In other words the weakness of gravity in this approach follows from the fact that only exponential tail of the full graviton's wave function can be seen by four-dimensional observer.

In a broader prospective, both these scenarios reduce hierarchy of the mass scales to some geometrical hierarchy. However, in functional terms, (1) and (2) are quite different: while the power law (1) makes L_P small introducing some large artificial scale R - extra dimension size: $L/L_P \sim (R/L)^{n/2}$; expression (2) corresponds to exponential increase: $L/L_P \sim \exp\gamma$, where huge number L/L_P is mapped into not so huge number γ. The physical reason for appearance of the exponent in [5] is quantum tunneling of gravitons.

Recently the extra-dimensional approach to the hierarchy problem has been given a new interpretation in [7] (and further developed in [8, 9]). It is based on the following physical idea (also discussed in different contexts in [10, 11, 12, 13, 14]): suppose that above some scale L there is a dramatic rise of multiplicity, i.e. the number of relevant degrees of freedom (d.o.f.) N becomes huge at energies $E \gtrsim 1/L$. Then one can argue, both on perturbative and on nonperturbative grounds, that the number of stable species with typical mass M must not exceed the following bound:

$$NM^2 \lesssim M_P^2 \tag{3}$$

up to some logarithmic corrections. Another incarnation of the same idea takes a single particle of mass M carrying exactly conserved quantum number of periodicity N, for example, Z_N gauge symmetry charge. Then the gravitational cutoff in such theories goes down to $M_P/\sqrt{N}$ and can be lowered to a TeV scale (thus solving the hierarchy problem, or, at least, giving it a completely new prospective) if one takes N of the order 10^{32}.

The bound (3) can be given a natural interpretation from Sakharov's induced gravity point of view ([15], see [16, 17, 18] for introduction into the subject). Indeed, typical contribution to Einstein-Hilbert gravitational Lagrangian from one-loop effective action of matter

particles in curved space can be written as:

$$M_P^2 = \frac{1}{G} = \frac{1}{G_0} - \frac{1}{2\pi}\mathrm{Tr}_{\,s}\left[M^2 - \mu^2 \log\left(\frac{M^2}{\mu^2}\right)\right] \tag{4}$$

where G_0 is "bare gravitational constant" (taken to be equal to infinity in original Sakharov's approach), trace $\mathrm{Tr}_{\,s}$ is taken over the spectrum with the corresponding numerical factors accounting for particle content of the theory and M is ultraviolet cutoff. If there are N particle species of a given type[2] in the theory, the trace scales as $-\mathrm{Tr}_{\,s}[..] \propto N$. Then, making original Sakharov's one-loop dominance assumption, one has from (4): $1/G \sim NM^2$, i.e. just eq.(3). The physical interpretation of this result is clear. The rigidity of space, i.e. its resistance against an attempt to curve it is proportional to the number of particle species living in quantum vacuum in this space because the curvature costs energy. Roughly speaking, more different particle types the theory contains, weaker is the gravity in this theory,[3] i.e. rich and non-degenerate spectrum tends to wash space-time distortions out.

Using alternative language for the same physics one can relate the induced contributions to M_P to spectral density function for the trace of energy-momentum tensor [19]:

$$\frac{1}{G} = i\,\frac{\pi}{6}\int d^4x x^2 \langle 0|\mathcal{T}(T(x)T(0))|0\rangle \tag{5}$$

where $x^2 = x_0^2 - \vec{x}^2$ and $T(x) = T_\mu^\mu(x) - \langle 0|T_\mu^\mu(x)|0\rangle$. If Green's function $\langle 0|\mathcal{T}(T(x)T(0))|0\rangle$ scales at small distances, according to Wilson OPE, as $\langle \mathcal{O}_0\rangle/(x^2)^4$ (up to logarithms), the integral diverges quadratically and true UV cutoff corresponds to $M_P \cdot \langle \mathcal{O}_0\rangle^{-1/2}$. With extremely large $\langle \mathcal{O}_0\rangle$, proportional to the number of species, it can be as small as a few TeV.

It is clear at the same time that introduction of the numbers like 10^{32} in the fundamental theory does not look quite natural. One therefore is interested in models where this rise is emergent rather than postulated. The main goal of this paper is to discuss a scenario where this huge rise in number of relevant excitations appears in somewhat natural way. The key physical idea is a change of statistics relevant d.o.f. obey - from Bose/Fermi at low energies to infinite (quantum Boltzmann) one at high energies. The paper is organized in the following way. In section 2 we remind the reader, using conventional theories as examples, how ultraviolet cutoff in low energy theory "knows" about the number of d.o.f. at high energies. In section 3 qualitative explanation of our scenario is given, while section 4 is devoted to simple quantum mechanical model, illustrating its main features. This section also present a summary of quon statistics and related issues.

2 Phase transitions and multiplicity change

We see that the change with energy in the structure of d.o.f. is a crucial point in the picture discussed above. Therefore it is interesting to discuss theoretical schemes where this change

[2] Say, N light noninteracting scalars. Of course, if both bosons and fermions present in the theory, their contributions partly cancel each other.

[3] Again, leaving aside supersymmetric-like cancelations.

is built in theory structure dynamically. The simplest known field theoretical example is given by Nambu Jona-Lasinio model. Fundamental Hamiltonian of this theory contains N fermion d.o.f.: $H = H[\psi_i^\dagger, \psi_i]$, $i = 1...N$. If chiral symmetry is broken, relevant low energy d.o.f. are pseudoscalars: $H_{eff} = H[\Phi^\dagger, \Phi]$. The formal method to get H_{eff} out of H is well known bosonization trick à la Hubbard - Stratonovich:

$$H[\psi_i^\dagger, \psi_i] \to H_0[\Phi] + H_0[\psi_i^\dagger, \psi_i] + H_{int}[\Phi, \psi_i^\dagger, \psi_i] \tag{6}$$

with subsequent integration over fermions. The phenomenon of the number of d.o.f. reduction at low energies is encoded in the term $H_{int} \sim \Phi \cdot \psi_i^\dagger \psi_i$.

The same logic applies to large N_c QCD. Let us take $SU(N_c)$ Yang-Mills gauge theory with two flavors of light fundamental quarks u and d. There are $2N_c$ spinor degrees of freedom in the theory with 2 staying for the number of flavors and N_c for the number of colors. At low energies the relevant excitations are three light pseudoscalar pions π^+, π^-, π^0 and the corresponding lowest order effective Lagrangian reads:

$$L_{eff} = \frac{1}{4} F^2 \mathrm{Tr}\ \left(\partial_\mu U^\dagger \partial^\mu U\right) + \frac{1}{2} F^2 B \mathrm{Tr}\ \left(m \left(U + U^\dagger\right)\right) \tag{7}$$

where the $SU(2)$ matrix field U is expressed in terms of the pion fields $\vec{\pi}(x) = (\pi^1(x)\,,\, \pi^2(x),$ $\pi^3(x))$, mass matrix $m = \mathrm{diag}(m_u, m_d)$, and F is just pion decay constant (up to $\mathcal{O}(m)$ corrections). The Lagrangian rewritten in terms of the pion fields takes the following form (neglecting $\mathcal{O}(\vec{\pi}^4)$ terms):

$$L = (m_u + m_d) F^2 B + \frac{1}{2} (\partial \vec{\pi})^2 - \frac{1}{2} (m_u + m_d) B \vec{\pi}^2 \tag{8}$$

with the obvious assignment $m_\pi^2 = (m_u + m_d)B$ and Gell-Mann - Oakes - Renner relation [20]: $(m_u + m_d)\langle \bar{q} q \rangle = -F^2 m_\pi^2$. It is worth noticing that despite the Lagrangian (8) does not contain N_c explicitly, the low energy description as such is valid up to pion momenta smaller than ultraviolet cutoff $\Lambda \sim 4\pi F$. The latter is N_c-dependent quantity (one can formally remove N_c-dependence from the Lagrangian (7) by redefinition of the pion fields but we prefer not to do it). Indeed, taking into account that quark condensate scales linearly with N_c and $m_\pi \to$ const in large N_c limit, one has: $F^2 \sim N_c$. Thus induced contribution of pions to the gravitational constant according to (4) scales as[4]

$$\delta \left(\frac{1}{G}\right)_{pions} \sim \Lambda^2 \sim N_c \tag{9}$$

despite there are only three and not N_c pions in the theory. Notice also the change of statistics, common for the theories with nontrivial vacuum of this kind: the fundamental d.o.f. are fermions, while low energy d.o.f. are bosons. The meaning of (9) is the same as has been just discussed: the ultraviolet cutoff of the low energy theory encodes information about the number of degrees of freedom in the "fundamental" theory.[5] More degrees of freedom

[4]At large N_c pure gluon contribution which scales as N_c^2 starts to dominate, but it is not seen at low energies until Λ becomes of the order of the lowest glueball mass.

[5]For Λ larger than masses of other, non-Goldstone hadrons, they have to be included as well.

(associated with color in the considered example) the underlying theory contains, weaker its low-energy excitations interacts with the gravity. This is exactly original idea behind [7] and it is not surprising that (9) is nothing than (3) seen from a different prospective. Notice that the "minimal charge" hypothesis discussed in [21] as a consequence of "gravity is the weakest force" conjecture can be rephrased as a statement that Planck mass M_P in a theory with dynamical scale λ and coupling g is never smaller than λ/g. In large N_c limit $g \sim 1/\sqrt{N_c}$ and we come back directly to (3) and (9).

3 Infinite statistics and quons

The main problem with the mechanism of d.o.f. reduction at low energies discussed above is its weak linear character. In other words, one gets as many d.o.f. as have been introduced into high energy theory from the beginning. Thus the gap between $n \sim$ a few dozens experimentally observed d.o.f. at low energies and 10^{32} d.o.f. at TeV energies needed to solve the hierarchy problem seems rather artificial and in some sense replaces the mass hierarchy by even worse hierarchy with typical dimensionless parameter 10^{32}. It seems therefore reasonable to look at models with stronger than linear number of d.o.f. enhancement. In the present paper, developing earlier results [22], we discuss a quantum mechanical model where this goal is achieved by change of statistics relevant d.o.f. obey from conventional Bose/Fermi ones at low energies to the infinite (quantum Boltzmann) one at high energies. Thus the fundamental d.o.f. in our models are quons. The motivation for this choice and short summary of infinite statistics properties are presented below, but before we start systematic exposition it is useful to explain main qualitative aspects. A distinctive feature which makes quon theories different from conventional Bose/Fermi ones is the fact that typical quon Hamiltonian (even for free theory) contains interaction vertices of all orders. This is in contrast with the standard situation where correct choice of the vacuum and relevant excitations usually allows to separate quadratic free part of Hamiltonian from high order terms, describing interactions. Thus having quon fields $\mathcal{A}_i^\dagger$, $\mathcal{A}_i$ and Bose/Fermi effective low energy field $\mathcal{F}_j$ one could have in general instead of (6)

$$H_{int} \sim \mathcal{F}_j \cdot \Big[c_{11}\mathcal{A}_i^\dagger\mathcal{A}_i + \sum_P c_{12}^P \mathcal{A}_i^\dagger\mathcal{A}_k^\dagger\mathcal{A}_k\mathcal{A}_i + ...\Big] + \mathcal{F}_j\mathcal{F}_l \cdot \Big[c_{21}\mathcal{A}_i^\dagger\mathcal{A}_i + ...\Big] + ... \tag{10}$$

where $c_{\alpha\beta}^P$ are coefficients depending (for $\beta > 1$) on particular permutation of the fields $\mathcal{A}_i$ and the summation over permutations $\sum_P$ takes into account the fact that for quon fields $\mathcal{A}_i\mathcal{A}_k \neq \pm\mathcal{A}_k\mathcal{A}_i$. It is important that in general we do not expect to have any small parameter in the expansion (10), which could make lowest terms dominant. Therefore it is natural to expect that quon content of the true low energy ground state $|\Omega\rangle$ (which fixes the subset of dominant terms in (10)) is determined dynamically by the high energy quon Hamiltonian $H[\mathcal{A}_i^\dagger, \mathcal{A}_i]$. If one assumes that it is nontrivial superposition of n-quon states, one immediately gets stronger than exponential[6] in n degeneracy for each low energy d.o.f. described by $\mathcal{F}_i$. Speaking differently, each low energy d.o.f. $\mathcal{F}_i$ interacts with (or, using

[6] And purely factorial in particular case of all n quons being of different flavors.

more informal language, is made of) factorially large number of *different* quon d.o.f. represented by the products $\mathcal{A}_{i_1}\mathcal{A}_{i_2}...\mathcal{A}_{i_n}$. These d.o.f. are condensed at low energy phase with ordinary fermions/bosons playing the rôle of light excitations over this nontrivial vacuum, while they become relevant d.o.f. at high energies. In this way one can easily get exponential multiplicity, resulting in:

$$\delta\left(\frac{1}{G}\right) \sim g_n M_n^2 \tag{11}$$

where the factor g_n of combinatorial origin scales with n as $n!$ or even n^n for totaly symmetric vacuum state. Without intention to cook up numerical factors it is interesting to notice that numerically one gets M_n at TeV scale for $n = 29\pm1$. Since the number of all relevant d.o.f. in the SM at top quark mass scale is about one hundred, the "reduction" of hierarchy problem is even stronger than needed. It does not seem disappointing since the model is clearly too crude to pretend for quantitative predictions. Nevertheless we find the proposed pattern attractive since the only "large" number one has to introduce in this case is the number of low energy "flavors", which is assumed to govern the low energy vacuum degree of degeneracy. In the next section we present a quantum-mechanical model where the features just discussed are explicit.

4 Quantum mechanical model example

To make our discussion self-contained, let us remind some basic facts about quon theories and infinite statistics. The physical interest to theories with unconventional statistics is mainly motivated by arguments coming from physics of black holes. Consider composite system built of k free bosons with masses m_i placed into the volume V. In conventional theory this system satisfies Bose-Einstein statistics.[7] However this is not the case if one includes gravity into consideration. Indeed, for k or m_i or $1/V$ large enough in units of gravitational constant G a black hole forms. The latter is an object which satisfies neither Bose-Einstein nor Fermi-Dirac statistics [24]. In quantum statistical sense black hole is identified by external observer as an object with infinite number of internal states. In other words, ideal gas of black holes is a system, whose many-body wave function is neither totally symmetric nor antisymmetric with respect to the black holes permutation (another example of this kind is given by gas of D0-branes [25]). Thus one can say that heavy d.o.f. (i.e. the black holes) satisfy infinite statistics and coexist with light d.o.f. (i.e. ordinary bosons or fermions) interacting with them in the course of absorption and emission.

As is known [26] there are only three types of self-consistent statistics one is allowed to consider in four-dimensional field-theoretical framework: parabosonic and parafermionic statistics, including Bose-Einsteins and Fermi-Dirac ones, and quantum Boltzmann, or infinite statistics, characterized by the relation

$$a_i a_j^\dagger = \delta_{ij} \tag{12}$$

augmented by the Fock-state representation defining relation $a_i|0\rangle = 0$. Quantum fields satisfying (12) are conventionally called quons. The statistics (12) was introduced in math-

[7]Of course, the same is true for fermions and Fermi-Dirac statistics if k is odd.

ematical context in [27] and applied in noncommutative probability theory, large-N models, stochastic calculus etc (see, e.g. review [28]). The first discussion of quon statistics in field theoretical context is given in [29] and various aspects of quon theories have been analyzed in many subsequent papers (see, e.g. [30, 31]). In particular, physical interpretation of dark energy in infinite statistics language from holographic prospective is discussed in [32, 33, 34].

In quantum Boltzmann statistics m-particle state is constructed as

$$|\phi_m^P\rangle = (a_{i_1}^\dagger)^{k_1}(a_{i_2}^\dagger)^{k_2}...(a_{i_l}^\dagger)^{k_l}|0\rangle \tag{13}$$

with $k_1 + k_2 + ... + k_l = m$. By index P we denote the concrete permutation of the creation operators, which is necessary since different permutations correspond in general to different states. All states have positive norm and can be normalized to unity by the condition $\langle 0|0\rangle = 1$. The states created by any permutations of creation operators are orthogonal, i.e.

$$\begin{gathered}(a_i^\dagger..a_m^\dagger|0\rangle)^\dagger \cdot (a_i^\dagger..a_m^\dagger|0\rangle) = \langle 0|a_m..a_i a_i^\dagger..a_m^\dagger|0\rangle = 1 \\ (a_i^\dagger..a_k^\dagger..a_l^\dagger..a_m^\dagger|0\rangle)^\dagger \cdot (a_i^\dagger..a_l^\dagger..a_k^\dagger..a_m^\dagger|0\rangle) = 0 \ \text{ for any } \ k \neq l\end{gathered} \tag{14}$$

For particles of the type i one can define the number operator N_i such that

$$N_i|\phi_m^P\rangle = k_i|\phi_m^P\rangle \ \text{ and } \ [N_i, a_i]_- = -a_i \tag{15}$$

as

$$N_i = a_i^\dagger a_i + \sum_l a_l^\dagger a_i^\dagger a_i a_l + \sum_{l,m} a_m^\dagger a_l^\dagger a_i^\dagger a_i a_l a_m + ... \tag{16}$$

or, in compact notation, $N_i = a_i^\dagger a_i + \sum_j a_j^\dagger N_i a_j$. For free normal-ordered Hamiltonian one has $H_0 = \sum\limits_{i=1}^{n} \mathcal{E}_0^i N_i$. The condition (12) automatically makes any product of quon operators normally ordered. As is already pointed out, it is a distinctive property of quon statistics that free Hamiltonian is given by an infinite series in creation and annihilation operators (in contrast with conventional Bose or Fermi case where free Hamiltonian is or can be made quadratic). Nevertheless there is no rich dynamics in free quon theory due to (12). Indeed, any state of the kind (13) or superposition of such states is an eigenstate of the Hamiltonian with an eigenvalue $\sum\limits_{i=1}^{n} \mathcal{E}_0^i k_i$. This is nothing than the standard harmonic oscillator equidistant spectrum.

It can be mentioning as a side remark that the structure of Hilbert space of states discussed in the present paper can be realized in alternative ways, i.e. without introducing quon d.o.f. Let us refer the reader to recent paper [35], where the authors discuss momentum-dependent statistics resulted from κ-deformed Poincaré algebra. In this approach the momentum splitting between, e.g. two-particle boson states $|p_1, p_2\rangle_\kappa$ and $|p_2, p_1\rangle_\kappa$ is of the order of $|\vec{p}_1| \cdot |\vec{p}_2|/\kappa$ where κ has the meaning of Planck mass M_P. This could be interpreted in terms of additional planckian internal degrees of freedom, which stay unresolved for energies much smaller than M_P. It would be interesting to reconcile those approach with the induced gravity paradigm.

In all theories based on quantum mechanics (and in the SM in particular) any probability density must be symmetric with respect to exchange of identical particles. While in

nonrelativistic quantum mechanics this symmetrization principle can be either postulated or derived under some assumptions, in standard quantum field theory it directly follows from Bose or Fermi commutation relations between field creation and annihilation operators. In quon theories there is no a priori any symmetry of this type. For example, the states $a_1^\dagger a_2^\dagger|0\rangle$ and $a_2^\dagger a_1^\dagger|0\rangle$ are orthogonal and in free theory they are degenerate. It is reasonable to expect that in interacting theory this degeneracy is lifted and the simplest way to get this is to associate interactions with permutations. To be more concrete, we assume that symmetry breaking due to quon condensation takes place as described by the following Hamiltonian (compare with (16)):

$$H = E(1 - A_n^\dagger A_n + (A_n^\dagger)^2(A_n)^2 + ...) \tag{17}$$

where A_n is a superposition of linearly independent n-quon annihilation operators:

$$A_n = (\alpha_n^P \cdot a_{i_1} a_{i_2} ... a_{i_n} + \text{permutations}) \tag{18}$$

where for simplicity all indices are taken to be different. The energy scale E is assumed to be large in the sense specified below. The coefficients α_n^P are chosen in such a way that the operators A_n obey the same infinite statistics as a_i does: $A_n A_n^\dagger = 1$. For totally symmetric combination it corresponds to permutation-independent $\alpha_n^P = 1/\sqrt{n!}$. It is worth noting that it is not necessary to take n in (17) equal to the number of quon species n or to omit terms with coinciding indices, since only general properties of H discussed below are important, not particular choice of A_n.

With the choice (17) the vacuum state at low energies is given by

$$|\Omega\rangle = A_n^\dagger|0\rangle \tag{19}$$

with $H|\Omega\rangle = 0$. Notice that $|\Omega\rangle$ is not a vacuum for original "high-energy" quon d.o.f. in the sense that $a_i|\Omega\rangle \neq 0$. On the other hand it should be the vacuum for low energy d.o.f.: $f_i|\Omega\rangle = 0$. The latter requirement can be reformulated if one introduces instead of low-energy excitations f_i over non-trivial vacuum $|\Omega\rangle$ composite operators $F_i^P = f_i \otimes a_{i_1}^\dagger a_{i_2}^\dagger ... a_{i_n}^\dagger$ acting on unbroken vacuum $|0\rangle$, where the index P reminds that different permutations of quon operators correspond, in general, to different excitations. Then from (18) and (19) one has

$$\sum_P \alpha_n^P F_i^P|0\rangle = 0 \tag{20}$$

The crucial point now is concrete mechanism how (20) is realized. Roughly speaking, in the effective theory language this depends on the way excitations f_i are made of excitations a_i. There are known explicit ways how to construct conventional fermion and boson operators out of quon ones [36], but what is important for us here is a structure of the sum (20). We consider two extreme scenarios: 1) all terms but one are linearly independent and 2) $F_i^P|0\rangle = 0$ for each transposition P. In the former case there is a unique non-degenerate vacuum in the theory and it is given by $|\Omega\rangle$. In the latter case each state $|\phi_n^P\rangle = a_{i_1}^\dagger ... a_{i_n}^\dagger|0\rangle$ (and any their linear combination) is annihilated by f_i: $f_i|\phi_n^P\rangle = 0$. So the space of eigenstates of the Hamiltonian (17) has the following structure in this case:

$$|\Omega\rangle = \sum_P \alpha_n^P|\phi_n^P\rangle \;\; ; \;\; |\omega^P\rangle = c^{P'}|\phi_n^{P'}\rangle + c^P|\phi_n^P\rangle \tag{21}$$

where P' is some arbitrarily chosen but fixed transposition different from P and coefficients c^P are chosen to have $\alpha_n^P c^P + \alpha_n^{P'} c^{P'} = 0$ and $\langle \omega^P | \omega^P \rangle = 1$. For permutation-invariant $\alpha_n^P = 1/\sqrt{n!}$ one has $c^{P'} = -c^P = 1/\sqrt{2}$. The vacuum has zero energy by construction: $\langle \Omega | H | \Omega \rangle = 0$ while all states $|\omega^P\rangle$ are degenerate and separated from the vacuum by the gap E: $\langle \omega^P | H | \omega^P \rangle = E$.

Thus we arrive to the following physical picture in this model. There is a ground state $|\Omega\rangle$ with n low energy excitations obeying conventional statistics over it. The corresponding masses m_i are assumed to be small at the scale of E. There are also $(n! - 1)$ states $|\omega^P\rangle$ with the excitations over them realized by the same n different Bose/Fermi operators, which are all considerably heavier (since E is assumed to be large). The high energy part of this tower is unseen in low energy processes characterized by the energies much smaller that E with the only exception: it dominantly contributes to the low energy gravitational constant making it extremely small.

To summarize, one possible way to explain why gravity is so weak in typical particle physics units is to assume that there is huge number of non-SM gravitationally interacting d.o.f., which are either quite heavy or do not interact with SM particles. It seems possible to explain this rise by change of statistics relevant degrees of freedom obey at high energies from Bose/Fermi to infinite one. In this scenario one gets typical scaling relation between true ultraviolet cutoff scale of low energy effective theory M and inverse gravitational constant seen by low energy observer (which is Planck mass M_P) of the following form:

$$M_P^2 \sim g_n M^2 \tag{22}$$

where the factor g_n is of combinatorial origin and it scales with n as $n!$ or even n^n where n is the number of low energy d.o.f. We presented simple quantum mechanical model illustrating this pattern, which we believe is typical for any quon theory with nontrivial vacuum.

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